POLARISATION

Theory of Production of Plane, Circularly and Elliptically Polarised Light

- As discussed earlier, the light that has unidirectional vibrations is known as plane polarised light or linearly polarised light.
- When two plane polarised light waves are allowed to superimpose, and the resultant electric vector rotates in such a way that its tip traces a circle, the resultant light is known as circularly polarised light.
- However, if the magnitude of the resultant electric vector varies periodically during its rotation and its tip traces an ellipse, then the resultant light is called elliptically polarised light.

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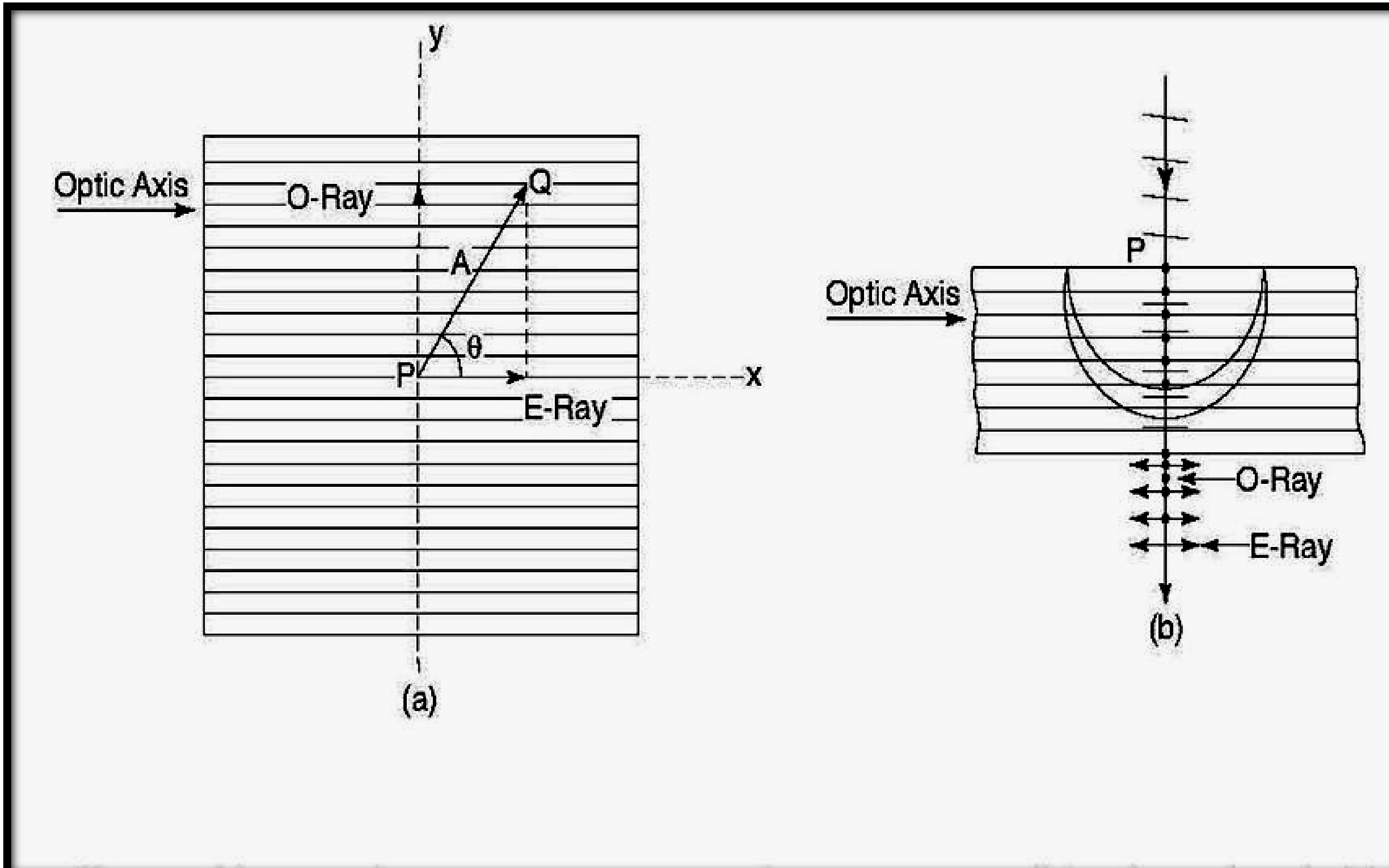

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- Let us consider a calcite crystal such that its refracting faces are cut parallel to its optic axis.
- Further we consider that the linear vibrations (amplitude A) in the incident light are along the direction PQ that makes an angle θ with the optic axis.
- (Fig. a). Under this situation, the incident plane polarised light of amplitude *A spits into two components A cos θ (E-ray) and A sin θ (O-ray).*
- *These components constitute E-ray and O-ray in view of their vibrations parallel to the optic axis and perpendicular to it, respectively.*
- *As per Huygens'* theory, the E-ray and O-ray travel in the same direction (Fig. b) with different velocities.
- Since calcite is a negative crystal, the velocity of E-ray will be greater than that of O-ray.
- Hence, a phase difference φ is introduced between them after traveling through the plate.

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In view of the incident light wave as $A \sin \omega t$, we can represent the E-ray along the optic axis as

$$x = A \cos\theta \sin(\omega t + \phi) \quad \text{(i)}$$

Similarly, the O-ray along Y-axis will be

$$y = A \sin\theta \sin \omega t \quad \text{(ii)}$$

Now assuming $A\cos\theta$ and $A \sin\theta = b$, we get

$$x = a \sin(\omega t + \phi) \quad \text{(iii)}$$

$$y = b \sin \omega t \quad \text{(iv)}$$

From Eq. (iv), we have

$$\sin \omega t = \frac{y}{b} \quad \text{(v)}$$

$$\text{and } \cos \omega t = \sqrt{1 - \frac{y^2}{b^2}} \quad \text{(vi)}$$

Now from Eq. (iii), we get

$$\frac{x}{a} = \sin \omega t \cos\phi + \cos \omega t \sin\phi \quad \text{(vii)}$$

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Putting the values of sin ωt and cos ωt from Eqs. (v) and (vi) in the above equation, we have

$$\frac{x}{a} = \frac{y}{b}\cos\phi + \sqrt{1-\frac{y^2}{b^2}}\ \sin\phi$$

$$\text{or } \frac{x}{a} - \frac{y}{b}\cos\phi = \sqrt{1-\frac{y^2}{b^2}}\ \sin\phi \qquad \text{(viii)}$$

On squaring both sides, Eq. (viii) we get

$$\left[\frac{x}{a} - \frac{y}{b}\cos\phi\right]^2 = \left[1-\frac{y^2}{b^2}\right]\sin\phi$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\phi - \frac{2xy}{ab}\cos\phi = \left[1-\frac{y^2}{b^2}\right]\sin^2\phi$$

$$\text{or } \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2\phi \qquad \text{(ix)}$$

This is the general equation of an ellipse.

Special Cases: Since the phase difference ϕ between the ordinary and extraordinary rays depend upon the thickness of the plate, we will discuss below the different cases on the basis of this thickness t.

Case-I: If the thickness of the plate is such that it introduces a phase difference of $\phi = 0, 2\pi, 4\pi, \ldots$ between O-ray and E-ray, then $\sin\phi = 0$ and $\cos\phi = 1$. Therefore, Eq. (ix) becomes,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

$$\text{or } \left[\frac{x}{a} - \frac{y}{b}\right]^2 = 0$$

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$$\text{or } y = \frac{b}{a}x \qquad \text{(x)}$$

This is the equation of straight line having the slop $\left(\frac{b}{a}\right)$ and passing through the origin Fig. 1 (a). This concludes that the light emerging through the plate is plane polarised.

Case-II: If the thickness of the plate is such that $\phi = \pi, 3\pi, 5\pi, \ldots$, then $\sin\phi = 0$ and $\cos\phi = -1$. Therefore, Eq. (ix) attains the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

or

$$\left[\frac{x}{a} + \frac{y}{b}\right]^2 = 0$$

$$y = \frac{-b}{a}x \qquad \text{(xi)}$$

This is again an equation of straight line having the slope $\left(-\frac{b}{a}\right)$ (Fig. 1 b). So we will have again the emergent light as plane polarised light.

Case-III: If the thickness of the plate is such that $\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$....., then $\sin\phi = 1, \cos\phi = 0$

Eq. (ix) attains the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{(xii)}$$

This is the equation of an ellipse with its axis along x and y directions (Fig. 1 c). Therefore, the emergent light will be elliptically polarised light.

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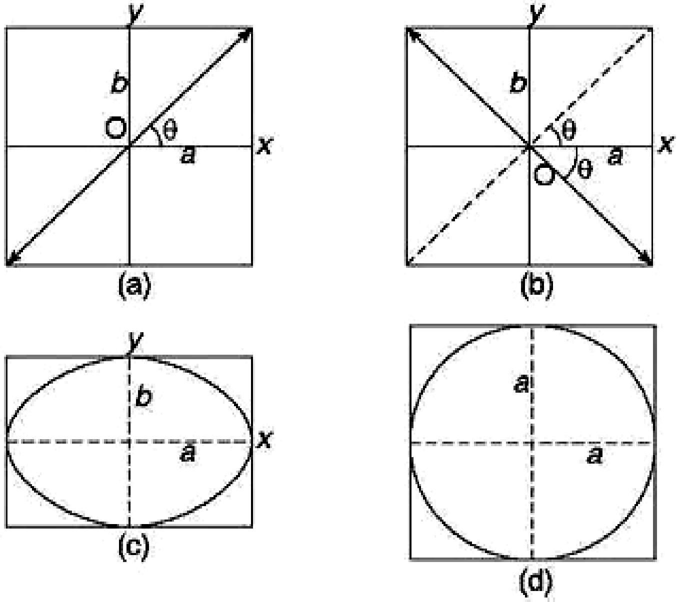
Case-IV: If $a = b$ and ϕ satisfies the condition of Case-III

$$x^2 + y^2 = a^2$$

This is the equation of a circle of radius a. Thus, the emergent light will be circularly polarised light if the plate introduces a phase change of

$$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \text{ etc.}$$



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From the above discussion it is clear that the plane and circularly polarised lights are the special cases of an elliptically polarised light which is obtained by the superposition of two plane polarised lights.